

# Monad constructions of omalous bundles

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## Abstract

We consider a particular class of holomorphic vector bundles relevant for supersymmetric string theory, called *omalous*, over nonsingular projective varieties. We use monads to construct examples of such bundles over 3-fold hypersurfaces in  $\mathbb{P}^4$ , complete intersection Calabi-Yau manifolds in  $\mathbb{P}^k$ , blow-ups of  $\mathbb{P}^2$  at  $n$  distinct points, and products  $\mathbb{P}^m \times \mathbb{P}^n$ .

## 1 Introduction

Let  $X$  be a nonsingular projective variety,  $TX$  be its tangent bundle and  $\omega_X$  its canonical line bundle. This paper is dedicated to the study of the following class of holomorphic vector bundles.

**Definition 1.1.** *A holomorphic vector bundle  $\mathcal{E} \rightarrow X$  is called omalous if it satisfies the following conditions:*

- $c_2(\mathcal{E}) = c_2(TX)$
- $\det(\mathcal{E}^*) \simeq \omega_X$

Recall also that a holomorphic vector bundle  $\mathcal{E} \rightarrow X$  is *slope (semi-)stable* with respect to a chosen polarization  $\mathcal{O}_X(1)$  of  $X$ , i.e., for every proper subsheaf  $\mathcal{F}$  of  $\mathcal{E}$  the inequality  $\frac{\deg(\mathcal{F})}{rk\mathcal{F}}(\leq) < \frac{\deg(\mathcal{E})}{rk\mathcal{E}}$  is satisfied.

The nomenclature comes from the fact that the matching of the first and second Chern classes of  $\mathcal{E}$  and  $TX$  is the usual Green-Schwarz anomaly cancellation condition in heterotic string theory [8, 12]; hence such bundles are *not anomalous*, that is *omalous* (Josh Guffin attributes this terminology to Ron Donagi, see the footnote in the first page of [13]).

Such bundles have a long history in the string literature. They appeared in the attempt at compactifying superstring theory to a theory on a  $M^4 \times X$ , (where  $X$  is complex compact Calabi-Yau 3-fold and  $M^4$  is a flat Minkowski

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\*Supported by FAPESP fellowship and grant 2009/18249-0

†Partially supported by the CNPq grant number 305464/2007-8 and FAPESP grant number 2005/04558-0.

space) with an unbroken  $N = 1$  supersymmetry in four dimensions. This was done by arguing perturbatively in terms of the string coupling constant as in [8, 20] and [12].

In the late nineties, arguments about compactifications using non-perturbative vacua in heterotic string theory were given by introducing five-branes as done by Donagi et. al. in [9]. In this context, the general formula for anomaly cancelation is given by

$$c_2(\mathcal{E}) - c_2(TX) = [W],$$

where  $[W]$  is the cohomology class of a four-form on the five-brane, and by Poincaré duality it corresponds to the class of an effective curve in the five-brane [9, Section 2].

More recently, the motivation to consider such bundles comes from two sources. First, the omality conditions are necessary to the construction of a *quantum sheaf cohomology* for the bundle  $\mathcal{E}$ , c.f. [13, Section 2] and [14]. The quantum sheaf cohomology of a bundle  $\mathcal{E} \rightarrow X$  is a generalization of the quantum cohomology of  $X$ , and consists of the structure of a Frobenius algebra on

$$QH^\bullet(\mathcal{E}) := \oplus_{p,k} H^p(X, \wedge^k \mathcal{E}^*) \otimes \mathbb{C}[[q]]$$

with product and bilinear pairing induced by the three-point correlation functions in a  $(0, 2)$  supersymmetric nonlinear sigma model; see [13, 14] and the references therein for further details.

Second, it was shown by Andreas and Garcia-Fernandez in [1] that stable omalous bundles over Calabi-Yau 3-folds admit solutions of the *Strominger system*, which is a system of coupled partial differential equations defined over a compact complex manifold relevant in heterotic string theory, c.f. [1] and the references therein.

The simplest example of omalous bundles are  $TX \oplus \mathcal{O}_X^{\oplus k}$  and its deformations; a few other examples were considered in [1, 6, 10, 14]. Our goal is to construct more examples of (stable) omalous bundles over various choices for  $X$  using *monads*.

We remark that in the attempt of giving phenomenological models from heterotic compactifications, particular *monads* have been used in the physics literature such as in [2, 3, 4, 5, 19]; we emphasize however that what is called a monad in [2, 3, 4] does not coincide with the usual definition in the mathematical literature.

Recall that a *monad* on  $X$  is a complex of locally free sheaves

$$M_{\bullet} : M_0 \xrightarrow{\alpha} M_1 \xrightarrow{\beta} M_2$$

such that  $\beta$  is locally right-invertible,  $\alpha$  is locally left-invertible. The (locally free) sheaves  $K := \ker \beta$ ,  $Q := \operatorname{coker} \alpha$  and  $E := \ker \beta / \operatorname{Im} \alpha$  are called, respectively, the *kernel*, *cokernel* and *cohomology* of  $M_{\bullet}$ .

In what follows, we provide examples of omalous bundles over 3-fold hypersurfaces in  $\mathbb{P}^4$ , complete intersection Calabi-Yau manifolds in  $\mathbb{P}^k$  ( $k = 4, 5, 6, 7$ ), blow-ups of  $\mathbb{P}^2$  at  $n$  distinct points, and products  $\mathbb{P}^m \times \mathbb{P}^n$ . All of these examples arise as cohomology, kernel, or cokernel of particular monads over these manifolds. We hope that such examples will be relevant for a deeper understanding of both quantum sheaf cohomology, the Strominger system and supersymmetric string theory.

**Acknowledgments** This paper would not exist if we had not heard Josh Guffin's and Mario Garcia-Fernandez's excellent talks during the Second Latin Congress on Symmetries in Geometry and Physics; we thank them for useful conversations during the conference. We also thank Rosa Maria Miró-Roig for her comments on the first version of this paper.

## 2 Stable omalous bundles over 3-fold hypersurfaces in $\mathbb{P}^4$

Let  $X$  be the non-singular quintic 3-fold in  $\mathbb{P}^4$ , and consider the following monad:

$$0 \longrightarrow \mathcal{O}_X(-1)^{\oplus 10} \xrightarrow{\alpha} \mathcal{O}_X^{\oplus 22} \xrightarrow{\beta} \mathcal{O}_X(1)^{\oplus 10} \longrightarrow 0. \quad (2.1)$$

The existence of the monads (2.1) is explicitly guaranteed by the construction in [16, Section 3]; its cohomology bundle  $\mathcal{E}$  is a rank 2 bundle with Chern classes  $c_1(\mathcal{E}) = 0$  and  $c_2(\mathcal{E}) = 10 \cdot H^2$ , where  $H = c_1(\mathcal{O}_X(1))$  is the ample generator of the Picard group of  $X$ . This bundle is stable by [16, Main Theorem]. Moreover  $c_2(\mathcal{E}) = c_2(TX)$  and  $\det(\mathcal{E}^*) \cong \mathcal{O}_X = \omega_X$  since the quintic 3-fold in  $\mathbb{P}^4$  is Calabi-Yau. Hence  $\mathcal{E}$  is a stable omalous bundle over  $X$ .

Let us consider now a non-singular 3-fold  $X_d$  of degree  $d$  in  $\mathbb{P}^4$ . One can show that

$$c_1(TX_d) = (5 - d) \cdot H \text{ and}$$

$$c_2(TX_d) = (d^2 - 5d + 10) \cdot H^2.$$

Let  $\mathcal{E}$  be a rank 3 linear bundle, that is, the cohomology of a linear monad of the form

$$0 \longrightarrow \mathcal{O}_{X_d}^{\oplus(c+l)}(-1) \xrightarrow{\alpha} \mathcal{O}_{X_d}^{\oplus(3+2c+l)} \xrightarrow{\beta} \mathcal{O}_{X_d}^{\oplus c}(1) \longrightarrow 0. \quad (2.2)$$

Then  $c_1(\mathcal{E}) = l \cdot H$  and  $c_2(\mathcal{E}) = [\frac{l}{2}(l+1) + c] \cdot H^2$ .

**Proposition 2.1.** (i) *The cohomology bundle  $\mathcal{E}$  of the linear monad (2.2) is omalous for every odd integer  $k \geq 7$ , such that  $(d, l, c)$  are given by*

$$d(k) = \frac{1}{2}(k-1), \quad l(k) = \frac{1}{2}(11-k), \quad c(k) = \frac{1}{8}(k^2 - 41).$$

(ii) *Furthermore the omalous bundle  $\mathcal{E}$  is stable for  $(d, l, c) = (3, 2, 1), (4, 1, 5)$  and semi-stable for  $(d, l, c) = (5, 0, 10)$ .*

**Proof.** (i) The conditions for which  $\mathcal{E}$  is omalous are given by  $c_1(\mathcal{E}) = c_1(TX)$  and  $c_2(\mathcal{E}) = c_2(TX)$ . In this case, one must have

$$\begin{cases} d^2 - 5d + 10 = c + \frac{1}{2}(5-d)(6-d) \\ l = 5 - d \end{cases}$$

Thus one must look for positive integer solutions  $d(c)$  of the quadratic equation  $d^2 + d - (10 + 2c) = 0$ , i.e.,  $d = \frac{1}{2}(-1 + \sqrt{41 + 8c})$ . For every odd integer  $k \geq 7$ , it is easy to verify that  $d(k) = \frac{1}{2}(k-1)$  are the desired roots, and the corresponding value for  $c$  is  $c(k) = \frac{1}{8}(k^2 - 41)$ .

(ii) The stability part follows from [17, Theorem 7]. If  $d = 5$  then  $l = 0$  and  $c = 10$ , Then  $\mathcal{E}$  is an instanton bundle, and by [17, Theorem 3], it is semi-stable.

□

**Remark 2.2.** The existence of the monads (2.2) is a consequence of Flyostad's Theorem for monads on  $\mathbb{P}^4$ . More precisely, the Main Theorem of [11] implies that the degeneration locus of a generic monad of the form

$$0 \longrightarrow \mathcal{O}_{\mathbb{P}^4}^{\oplus(c+l)}(-1) \xrightarrow{\alpha} \mathcal{O}_{\mathbb{P}^4}^{\oplus(3+2c+l)} \xrightarrow{\beta} \mathcal{O}_{\mathbb{P}^4}^{\oplus c}(1) \longrightarrow 0.$$

is zero dimensional. Its restriction to a generic hypersurface  $X_d$  is precisely (2.2), hence its cohomology yields a vector bundle over it.

### 3 Omalous bundles on complete intersection Calabi-Yau 3–folds

Let  $X$  be a complete intersection Calabi-Yau 3–fold in  $\mathbb{P}^n$ . There are only five such cases, namely:

- A quintic in  $\mathbb{P}^4$ .
- In  $\mathbb{P}^5$ , either the intersection of two cubics or the intersection of a quadric and a quartic.
- In  $\mathbb{P}^6$  the intersection of two quadrics with a cubic.
- In  $\mathbb{P}^7$  the intersection of four quadrics.

One can write  $X = \cap_i^l X_i$  where the  $X_i$ 's are given as in the list above and  $l = \text{codim}_{\mathbb{P}^n}(X)$ . Moreover one has the following short exact sequences

$$0 \longrightarrow TX_i \longrightarrow T\mathbb{P}^n|_{X_i} \longrightarrow \mathcal{N}_i \longrightarrow 0$$

$$0 \longrightarrow \mathcal{O}_{\mathbb{P}^n}(-d_i) \longrightarrow \mathcal{O}_{\mathbb{P}^n} \longrightarrow \mathcal{O}|_{X_i} \longrightarrow 0$$

where the normal bundle  $\mathcal{N}_i$  to  $X_i$  is simply the invertible sheaf  $\mathcal{O}_{\mathbb{P}^n}(d_i)$  since each of the  $X_i$  is a hypersurface of degree  $d_i$  in  $\mathbb{P}^n$ . Using these sequences one can easily prove that the Chern Class of the tangent bundle  $TX$ , to  $X$ , is given by the formula

$$C(TX) = \frac{(1+h)^{n+1}}{\prod_{i=1}^l (1+d_i h)}$$

where  $h = c_1(\mathcal{O}_{\mathbb{P}^n}(1))$ . From the condition  $c_1(TX) = 0$ , it follows that

$$c_2(TX) = \frac{1}{2}[(\sum_{i=1}^l d_i^2) - (n+1)]h^2.$$

**Proposition 3.1.** *Let  $\mathcal{E}$  be a rank 2 bundle on a complete intersection Calabi-Yau 3–fold  $X$  given by the cohomology of the following monad*

$$M : \quad 0 \longrightarrow \mathcal{O}_X(-1)^{\oplus c} \xrightarrow{\alpha} \mathcal{O}_X^{\oplus 2+2c} \xrightarrow{\beta} \mathcal{O}_X(1)^{\oplus c} \longrightarrow 0.$$

with  $c$  given according to the following table:

| $X$                      | $l$ | $(d_1, \dots, d_l)$ | $c = \frac{1}{2}[(\sum_{i=1}^l d_i^2) - (n+1)]$ |
|--------------------------|-----|---------------------|---|
| $X \subset \mathbb{P}^4$ | 1   | 5                   | 10  |
| $X \subset \mathbb{P}^5$ | 2   | (3, 3)              | 6   |
| $X \subset \mathbb{P}^5$ | 2   | (4, 2)              | 7   |
| $X \subset \mathbb{P}^6$ | 3   | (2, 2, 3)           | 5   |
| $X \subset \mathbb{P}^7$ | 4   | (2, 2, 2, 2)        | 4   |

Then  $\mathcal{E}$  is a stable and omalous.

**Proof.** Follows from the Main Theorem in [16] and the calculations above.  $\square$

## 4 Omalous bundles on multi-blow-ups of the projective plane

Let  $\pi : \tilde{\mathbb{P}}(n) \longrightarrow \mathbb{P}^2$  be the blow-up of the projective plane at  $n$  distinct points. Its Picard group is generated by  $n+1$  elements, namely:  $\text{Pic}(\tilde{\mathbb{P}}(n)) = \oplus_{i=1}^n E_i \mathbb{Z} \oplus H \mathbb{Z}$ , where every  $E_i$  is an exceptional divisor and  $H$  is the divisor given by the pull-back of the generic line in  $\mathbb{P}^2$ . The intersection form is given by:  $E_i^2 = -1$ ,  $E_i \cdot E_j = 0$  for  $i \neq j$ ,  $E_i \cdot H = 0$  and  $H^2 = 1$ . The canonical divisor of the surface  $\tilde{\mathbb{P}}(n)$  is given by  $K_{\tilde{\mathbb{P}}(n)} = -3H + \sum_{i=1}^n E_i$ . In terms of line bundles, a divisor of the form  $D = pH + \sum_{i=1}^n q_i E_i$  has the associated line bundle  $\mathcal{O}(D) = \mathcal{O}(p, \vec{q}) = \mathcal{O}(pH) \otimes \mathcal{O}(q_1 E_1) \otimes \cdots \otimes \mathcal{O}(q_n E_n)$  where  $\vec{q} = (q_1, \dots, q_n)$ .

Let  $H^2 \in H^4(\tilde{\mathbb{P}}(n), \mathbb{Z})$  be the fundamental class of  $\tilde{\mathbb{P}}(n)$ . For a torsion-free sheaf  $\mathcal{E}$ , of Chern character  $ch(\mathcal{E}) = r + (aH + \sum_{i=1}^n a_i E_i) - (k - \frac{a^2 - |\vec{a}|^2}{2})H^2$ , twisted by a line bundle  $\mathcal{O}(p, \vec{q})$  the Riemann-Roch formula is given by:

$$\begin{aligned} \chi(\mathcal{E}(p, \vec{q})) = & -[k - \frac{a}{2}(a+3) + \frac{1}{2}\sum_{i=1}^n a_i(a_i-1)] + \frac{r}{2}[(p+1)(p+2) - \sum_{i=1}^n q_i(q_i-1)] \\ & + [ap - \sum_{i=1}^n a_i q_i]. \end{aligned}$$

Note that the notations used through this section are the ones given in [15].

Omalous bundles  $\mathcal{E}$  on  $\tilde{\mathbb{P}}(n)$  are given in this case by the conditions:

$$\det(\mathcal{E}^*) = \omega_{\tilde{\mathbb{P}}(n)} = \mathcal{O}_{\tilde{\mathbb{P}}(n)}(-3, \vec{1}), \quad c_2(\mathcal{E}) = c_2(T\tilde{\mathbb{P}}(n)) = (3+n) \cdot H^2$$

Moreover, suppose that the direct image  $\pi_*(\mathcal{E})$ , of  $\mathcal{E}$ , is a normalized and semi-stable torsion free sheaf (in our case, normalized means that  $3 < r$ ). Then we have the following:

**Proposition 4.1.** *On a multi-blow-up  $\tilde{\mathbb{P}}(n)$  of the projective plane with  $n \geq 3$ , let  $\mathcal{E}$  be an omalous bundle of rank  $r > 3$  with semi-stable direct image  $\pi_*(\mathcal{E})$ . Then  $\mathcal{E}$  is the cohomology of the following monad*

$$M : \quad 0 \longrightarrow \oplus_{i=0}^n K_i(-1, E_i) \xrightarrow{\alpha} W \otimes \mathcal{O}_{\tilde{\mathbb{P}}(n)} \xrightarrow{\beta} \oplus_{i=0}^n L_i(1, -E_i) \longrightarrow 0$$

where we put  $E_0 := 0$  and  $K_i, L_i$  and  $W$  are vector space of dimensions

$$\dim K_i = \begin{cases} n & i = 0 \\ 2n-3 & \text{otherwise} \end{cases} \quad \dim L_i = \begin{cases} 2n-3 & i = 0 \\ 2n-4 & \text{otherwise} \end{cases}$$

and  $\dim W = 4n(n-1) - 3 + r$ .

**Proof.** The existence of the monad is guaranteed by [7, Proposition 1.10] since the direct image  $\pi_*(\mathcal{E})$  is semi-stable and normalized ( $r > 3$ ). The omality condition implies that the bundle  $\mathcal{E}$  has the following Chern character  $ch(\mathcal{E}) = r + (3H - \sum_{i=1}^n E_i) - \frac{3}{2}(n-1)H^2$ . The vector spaces in the monad are explicitly given by [7, Proposition 1.10]:  $K_0 = H^1(\tilde{\mathbb{P}}(n), \mathcal{E}^*(-1, 0))$ ,  $K_i = H^1(\tilde{\mathbb{P}}(n), \mathcal{E}(-1, 0))$  for  $i \neq 0$ , and  $L_0 = H^1(\tilde{\mathbb{P}}(n), \mathcal{E}(-1, 0))$ ,  $L_i = H^1(\tilde{\mathbb{P}}(n), \mathcal{E}(-1, E_i))$  for  $i \neq 0$ . Their dimensions follow by the Riemann-Roch Formula.  $\square$

## 5 Omalous bundles on $\mathbb{P}^n \times \mathbb{P}^m$

Let  $X = \mathbb{P}^n \times \mathbb{P}^m$ , with the natural projections

$$\begin{array}{ccc} & \mathbb{P}^n \times \mathbb{P}^m & \\ \pi_1 \swarrow & & \searrow \pi_2 \\ \mathbb{P}^n & & \mathbb{P}^m \end{array}$$

Its Picard group is generated by  $h_1 = \pi_2^* c_1(\mathcal{O}_{\mathbb{P}^n}(1))$  and  $h_2 = \pi_1^* c_1(\mathcal{O}_{\mathbb{P}^m}(1))$ , then  $Pic(X) = h_1\mathbb{Z} \oplus h_2\mathbb{Z}$ . The Chow ring of  $X$  is given by

$$A(X) = \mathbb{Z}[h_1, h_2] / (h_1^{n+1}, h_2^{m+1}).$$

Let  $L := \mathcal{O}_X(1, 1)$  be the ample line bundle associated to the ample divisor  $h_1 + h_2$ . For any sheaf  $\mathcal{F}$  on  $X$ , we define the degree of  $\mathcal{F}$  with respect to  $L$  as  $deg_L(\mathcal{F}) := c_1(\mathcal{F}) \cdot c_1(L)^{n+m-1}$ . Using the binomial formula one obtains  $c_1(L)^{n+m-1} = l(n, m)[h_1^{n-1} \cdot h_2^m + \frac{m}{n} h_1^n \cdot h_2^{m-1}]$  where  $l(n, m) = \frac{n(n+1) \cdots (n+m+1)}{m!}$ . It follows that if  $c_1(\mathcal{F}) = p \cdot h_1 + q \cdot h_2$ , then  $deg_L(\mathcal{F}) = l(n, m)[p + \frac{m}{n} q] \cdot h_1^n \cdot h_2^m$ . We define the  $L$ -slope  $\mu_L(\mathcal{F})$  of the sheaf  $\mathcal{F}$  by  $\mu_L(\mathcal{F}) := \frac{deg_L(\mathcal{F})}{rk(\mathcal{F})}$ , and will say that  $\mathcal{F}$  is  $L$ -(semi-)stable if for every subsheaf  $\mathcal{G} \subset \mathcal{F}$  the inequality  $\mu_L(\mathcal{G})(\leq) < \mu_L(\mathcal{F})$  is satisfied.

The tangent bundle of  $X$  is given by the following Euler sequence:

$$0 \longrightarrow \mathcal{O}_X^{\oplus 2} \longrightarrow \mathcal{O}_X^{\oplus n+1}(1, 0) \oplus \mathcal{O}_X^{\oplus m+1}(0, 1) \longrightarrow TX \longrightarrow 0,$$

from which one can easily compute the canonical bundle  $\omega_X = \mathcal{O}_X(-n-1, -m-1)$ , the first Chern class  $c_1(TX) = c_1(X) = (n+1) \cdot h_1 + (m+1) \cdot h_2$  and the second Chern class  $c_2(TX) = \frac{1}{2}(n+1)h_1^2 + \frac{1}{2}m(m+1)h_2^2 + (n+1)(m+1) \cdot h_1 \cdot h_2$ .

Now let us consider a rank  $(b+c-a)$ -bundle  $Q$  fitting in the following short exact sequence:

$$0 \longrightarrow \mathcal{O}_X^{\oplus a} \longrightarrow \mathcal{O}_X(1, 0)^{\oplus b} \oplus \mathcal{O}_X(0, 1)^{\oplus c} \longrightarrow Q \longrightarrow 0. \quad (5.1)$$

**Proposition 5.1.** *(i)  $Q$  is omalous for the values  $(b, c) = (n+1, m+1)$ .*

*(ii) The bundle  $Q$  is  $L$ -stable.*

**Proof.**

(i) From the exact sequence defining the bundle  $Q$  one can easily compute the Chern classes  $c_1(Q) = b \cdot h_1 + c \cdot h_2$  and  $c_2(Q) = \frac{b(b-1)}{2} \cdot h_1^2 + \frac{c(c-1)}{2} \cdot h_2^2 + bc \cdot h_1 \cdot h_2$ . The result follows by imposing the conditions  $c_1(Q) = c_1(X)$  and  $c_2(Q) = c_2(TX)$ .

(ii) Note that the twisted dual bundle  $Q^*(0, 1)$  fits in the following exact sequence

$$0 \longrightarrow Q^*(0, 1) \longrightarrow \mathcal{O}_X^{\oplus b}(-1, 1) \oplus \mathcal{O}_X^{\oplus c} \longrightarrow \mathcal{O}_X(0, 1)^{\oplus a} \longrightarrow 0. \quad (5.2)$$

By using the first statement in [18, Theorem 8], it follows that  $Q^*(0, 1)$  is  $L$ -stable, thus  $Q$  is also  $L$ -stable.

□

In particular, when  $a = 2$  and the omalous conditions are satisfied, then  $Q$  has exactly rank  $n+m$ , thus it is a deformation of the tangent bundle  $TX$ . Moreover it is easy to see that any deformation of  $TX$  is given as the last term bundle in the sequence (5.1), with  $a = 2$ ,  $b = n+1$  and  $c = m+1$ . Hence we have the following:

**Corollary 5.2.** *Any deformation of the tangent bundle  $TX$  of  $X = \mathbb{P}^n \times \mathbb{P}^m$  is  $L$ -stable.*

**Proof.** Follows from (i) in 5.1.

□



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